

$$0! = 1$$

$$f^{(k)}(a)$$

$$\frac{1}{k!} (x-a)^k$$

### Section 3

① a)  $f(x) = \sqrt{x}$ ,  $n=3$ ,  $a=1$

$$\begin{aligned} f(x) &= x^{1/2} & f(1) &= 1 \\ f'(x) &= \frac{1}{2}x^{-1/2} & f'(1) &= \frac{1}{2} \\ f''(x) &= -\frac{1}{4}x^{-3/2} & f''(1) &= -\frac{1}{4} \\ f'''(x) &= +\frac{3}{8}x^{-5/2} & f'''(1) &= \frac{3}{8} \end{aligned}$$

$$P_3(x) = \left(\frac{3}{8}\right)\left(\frac{1}{3}\right)(x-1)^3 + \left(-\frac{1}{4}\right)\frac{1}{2}(x-1)^2 + \left(\frac{1}{2}\right)(x-1) + 1$$

$$(P_3(x) = \frac{1}{16}(x-1)^3 - \frac{1}{8}(x-1)^2 + \frac{1}{2}(x-1) + 1) \quad \checkmark$$

$$\begin{aligned} b) \quad f(x) &= e^x & f(e) &= e^e \\ f'(x) &= e^x & f'(e) &= e^e \\ f''(x) &= e^x & f''(e) &= e^e \\ f'''(x) &= e^x & f'''(e) &= e^e \\ f''''(x) &= e^x & f''''(e) &= e^e \end{aligned}$$

$$P_4(x) = \frac{e^e}{4!}(x-e)^4 + \frac{e^e}{3!}(x-e)^3 + \frac{e^e}{2!}(x-e)^2 + e^e(x-e) + e^e$$

$$\begin{aligned} c) \quad f(x) &= (1-x^2)^{1/2} & f(0) &= 1 \\ f'(x) &= \frac{1}{2}(1-x^2)^{-1/2}(-2x) = \frac{-x}{(1-x^2)^{1/2}} & f'(0) &= 0 \\ f''(x) &= -\frac{1}{2}(1-x^2)^{-1/2} + \frac{1}{2}(1-x^2)^{-1/2}(+2x)(-x) & f''(0) &= -1 \\ f''(x) &= -\frac{1}{2}(1-x^2)^{-1/2} \left[ 1-x^2+x^2 \right] = \frac{-1}{(1-x^2)^{3/2}} \end{aligned}$$

$$P_2(x) = -\frac{1}{2}(x)^2 + 0x + 1$$

$$P_2(x) = -\frac{1}{2}x^2 + 1 \quad \checkmark$$

$$\textcircled{1} \quad f(x) = x^3 + 3x^2 - 2x + 8 \quad f(0) = 8$$

$$f'(x) = 3x^2 + 6x - 2 \quad f'(0) = -2$$

$$f''(x) = 6x + 6 \quad f''(0) = 6$$

$$f'''(x) = 6 \quad f'''(0) = 6$$

$$(P_3(x) = x^3 + 3x^2 - 2x + 8) \quad \checkmark$$

$$\textcircled{2} \quad a) \quad f(x) = \ln(x) \quad n=4 \quad f(1)=0$$

$$f'(x) = \frac{1}{x} = x^{-1} \quad a=1 \quad f'(1)=1$$

$$f''(x) = -x^{-2} \quad f''(1) = -1$$

$$f'''(x) = 2x^{-3} \quad f'''(1) = 2$$

$$f''''(x) = -6x^{-4} \quad f''''(1) = -6$$

$$(P_4(x) = -\frac{1}{4}(x-1)^4 + \frac{1}{3}(x-1)^3 - \frac{1}{2}(x-1)^2 + 1(x-1)) \quad \checkmark$$

$$\textcircled{3} \quad \textcircled{a} \quad f(x) = x^{-1/2} \quad f(4) = \frac{1}{2} \quad n=2$$

$$f'(x) = -\frac{1}{2}x^{-3/2} \quad f'(4) = -\frac{1}{16} \quad a=4$$

$$f''(x) = \frac{3}{4}x^{-5/2} \quad f''(4) = \frac{3}{128}$$

$$(P_2(x) = \frac{3}{256}(x-4)^2 - \frac{1}{16}(x-4) + \frac{1}{2}) \quad \checkmark$$

$$\textcircled{b} \quad f(x) = \cos(2x) \quad f(0) = 1 \quad n=3$$

$$f'(x) = -2\sin(2x) \quad f'(0) = 0 \quad a=0$$

$$f''(x) = -4\cos(2x) \quad f''(0) = -4$$

$$f'''(x) = +8\sin(2x) \quad f'''(0) = 0$$

$$P_3(x) = 0(x)^3 - \frac{4}{2}(x)^2 + 0(x) + 1$$

$$(P_3(x) = -2x^2 + 1) \quad \checkmark$$

C)  $f(x) = \cos(2x)$      $f(\pi/3) = -\frac{1}{2}$   
 $f'(x) = -2\sin(2x)$      $f'(-\pi/3) = +2 \cdot -\frac{\sqrt{3}}{2} = -\sqrt{3}$   
 $f''(x) = -4\cos(2x)$      $f''(-\pi/3) = -4 \cdot -\frac{1}{2} = 2$   
 $f'''(x) = 8\sin(2x)$      $f'''(-\pi/3) = 8 \cdot -\frac{\sqrt{3}}{2} = -4\sqrt{3}$

$n=3$   
 $a=-\pi/3$

$$P_4(x) = -\frac{2\sqrt{3}}{3}(x+\pi/3)^3 + (x+\pi/3)^2 + \sqrt{3}(x+\pi/3) - \frac{1}{2}$$

D)  $f(x) = \frac{1}{x} = x^{-1}$      $f(5) = \frac{1}{5}$   
 $f'(x) = -x^{-2}$      $f'(5) = -\frac{1}{25}$   
 $f''(x) = 2x^{-3}$      $f''(5) = \frac{2}{125}$   
 $f'''(x) = -6x^{-4}$      $f'''(5) = -\frac{6}{625}$

$n=3$   
 $a=5$

$$P_3(x) = -\frac{1}{625}(x-5)^3 + \frac{1}{125}(x-5)^2 - \frac{1}{25}(x-5) + \frac{1}{5}$$

⑤ shifted to  $x=1$

$\ln(1+x) = \ln(1-\boxed{-x})$     Replace  $(1-x)$  in Taylor  $P_n(x)$  for  
 $\ln(1-x) = \ln(1-\boxed{x})$   
 $\ln(x) = \ln(1-\boxed{(1-x)})$      $\ln(1-x)$ .

$f(-2) = \frac{-2}{5}$

⑦ @)  $f(x) = x^3 \sin(x)$      $a=0$     ⑦)  $f(x) = \frac{x}{1+x^2}$      $a=-2$

$$P_3(x) = 0$$

$$f'(x) = \frac{(1+x^2) - 2x(x)}{(1+x^2)^2} \quad f'(-2) = \frac{-3}{25}$$

B)  $f(x) = \frac{x}{1+x^2}$   
 $a=0$

$$f''(x) = \frac{-2x(1+x^2)^2 - 2(1+x^2)(2x)(1-x^2)}{(1+x^2)^4}$$

$$P_3(x) = x - x^3$$

$$f''(x) = \frac{-2x(3-x^2)}{(1+x^2)^3} \quad f''(-2) = \frac{4(-1)}{125} = \frac{-4}{125}$$

$$f'''(-2) = \frac{42}{625}$$

$$P_3(x) = \frac{7}{625}(x+2)^3 - \frac{2}{125}(x+2)^2 - \frac{3}{25}(x+2) - \frac{2}{5}$$

$$\textcircled{d} \quad f(x) = \tan x \quad a=0 \quad f(0)=0$$

$$f'(x) = \sec^2 x \quad f'(0)=1$$

$$f''(x) = 2\sec x \sec x \tan x \quad f''(0)=0$$

$$2\sec^2 x \tan x$$

$$f'''(x) = 4\sec x \sec x \tan x \cdot \tan x + \sec^2 x \cdot 2\sec^2 x$$

$$f'''(x) = 4\sec^2 x \tan^2 x + 2\sec^4 x$$

$$f'''(0)=2$$

$$P_3(x) = \frac{1}{3}x^3 + 0x^2 + 1x + 0$$

$$P_3(x) = \frac{1}{3}x^3 + x \quad \boxed{\checkmark}$$

$$\textcircled{11} \quad P_3(x) = (x+4)^3 + \frac{1}{2}(x+4)^2 + 0(x+4) + 2$$

$$P_3(-4,2) = 2.012 \quad \boxed{\checkmark}$$

$$P_2(x) = \frac{1}{2}(x+4)^2 + 2 \quad \boxed{\checkmark}$$

$$P_2(-4,2) = 2.02$$

(P<sub>3</sub>)

$$\textcircled{13} \quad f^{(n)}(0) = (-1)^n \cdot \frac{n^2+1}{n} \quad n \geq 1$$

$$f(0)=6$$

$$f'(0) = -1 \cdot 2 = -2$$

$$f''(0) = +1 \cdot \frac{5}{2} = 5/2$$

$$P_2(x) = \frac{5}{4}x^2 - 2x + 6 \quad \boxed{\checkmark}$$

$$P_3'(x) = -1 + 2(x+1) + 36(x+1)^2$$

$$P_3''(x) = 2 + 72(x+1)$$

$$\textcircled{15} \quad \textcircled{a} \quad f(-1) = \boxed{2} \quad \checkmark$$

$$\textcircled{b} \quad f'(-1) = P_3'(-1) = 36(0)^2 + 0 + -1$$

$$P_3'(-1) = \boxed{-1} \quad \checkmark \quad P_3'''(x) = 72$$

$$\textcircled{d} \quad f'''(-1) = \boxed{72} \quad \checkmark$$

$$\textcircled{c} \quad f''(4) = \text{Not enough info.} \quad \checkmark$$

$$\textcircled{17} \quad f(-3) = 8 \quad P_2(x) = 0(x+3)^2 + 1(x+3) + 8 \quad \boxed{\checkmark}$$

$$f'(-3) = 1$$

$$f''(-3) = 0$$

$$\text{when } x = -3 \quad \begin{cases} f(x) = -x & f'''(x) = 0 \\ f'(x) = -1 & f^4(x) = 0 \\ f''(x) = 0 \end{cases}$$

$$⑯ f(x) = |x|$$

when  
 $x=2$

$$\begin{cases} f(x) = x & \text{when } x \neq 2 \\ f'(x) = 1 & \\ f''(x) = 0 & \\ f'''(x) = 0 & \\ f^4(x) = 0 & \end{cases}$$

$$\begin{aligned} P_4(x) &= 2 + \frac{1}{1!}(x-2)^1 + \frac{0}{2!}(x-2)^2 + \dots + 0 \\ P_4(x) &= 3 + \frac{-1}{1!}(x+3)^1 + \frac{0}{2!}(x+3)^2 + 0 \\ P_4(x) &= x \quad \text{for } x = 2 \\ P_4(x) &= -x \quad \text{for } x = -3 \end{aligned}$$

$$⑰ f(x) = (1+x)^k$$

$$f'(x) = k(1+x)^{k-1}$$

$$f''(x) = k(k-1)(1+x)^{k-2}$$

$$f(0) = 1$$

$$f'(0) = k$$

$$f''(0) = k^2 - k$$

$$P_2(x) = \frac{(k^2 - k)}{2} x^2 + kx + 1$$

more!

⑯

$$f(x) = |x|$$

$$4\text{th} \quad P_4(x)$$

$$x = -3$$

$$x = 2$$

$$f(2) = 2$$

$$f'(2) = 1$$

$$f''(2) = 0$$

$$\downarrow 0$$

$$f(x) = x$$

$$f'(x) = 1$$

$$f''(x) = 0$$

$$f'''(x) = 0$$

$$f''''(x) = 0$$

$$P_4(x) = \frac{1(x-2)^1}{1!} + 2(x-2)^0$$

$$P_4(x) = x - 2 + 2 = \boxed{x} @ x = 2$$

$$P_4(x) = \boxed{-x} @ x = -3$$